B.Sc. 5th Semester (Honours) Examination, 2022 (CBCS)

Subject : Physics

Course : CC-XI

Quantum Mechanics & Applications

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Symbols and abbreviations have their usual meanings.

1. Answer *any five* of the following questions: 2×5=10

(a) The trial wavefunction of the one-dimensional infinite square well is given by

$$\Psi(x) = \begin{cases} Ax & if\ 0 \le x \le \frac{a}{2} \\ A(a-x) & if\ \frac{a}{2} \le x \le a \\ 0 & otherwise \end{cases}$$

sketch $\Psi(x)$ and determine the normalization constant A.

(b) Consider a system described by a state, which is a superposition of two orthonormal states Φ_1 and Φ_2 as $\Psi = \frac{\sqrt{3}}{2}\Phi_1 + \frac{1}{2}\Phi_2$. Also consider an ensemble of 50 identical systems each one of them in the state of Ψ. If measurements are done on all of them, how many systems will be found in each of the states Φ_1 and Φ_2?

(c) Find the spectroscopic notation for the ground state configuration of Aluminum $Al(Z = 13)$ and Scandium $(Z = 21)$.

(d) Differentiate between a weak-field Zeeman effect and the Paschen-Back effect in terms of the relative energy level spacings due to the spin-orbit effect and Zeeman effect.

(e) Which among the following wave functions represents physically acceptable functions: $f(x) = 3\sin\pi x, g(x) = 4 - |x|, h^2(x) = 5x$.

(f) What is Bohr magneton? Calculate its value.

(g) Why do all alkali atoms have qualitatively similar spectra?

(h) Consider a system of three non-interacting identical spin $\frac{1}{2}$ particle (each of mass m) that are in the same spin state $\left|\frac{1}{2}\frac{1}{2}\right\rangle$ and confined in one-dimensional infinite well of length a; $V(x) = 0$ for $0 < x < a$ & $V(x) = \infty$ for other values of x. (Assume $E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$). Determine energy of the ground state and the first excited state of the system.

2. Answer *any two* of the following questions: 5×2=10

(a) Given that $\widehat{p_r} = -i\hbar\left(\frac{\partial}{\partial r} + \frac{1}{r}\right)$. Find the uncertainty Δp_r in the ground state, $\Psi_0(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-\frac{r}{a_0}}$.

(b) (i) Show that the momentum operator $\widehat{p_x}$ is hermitian.

(ii) Show that the commutator $\left[\widehat{L_z}, \widehat{L_-}\right]$ is equal to $-\hbar\widehat{L_-}$, where all the symbols have their usual meaning. 2+3=5

(c) (i) Write a qualitative explanation of the fine-structure splitting of spectral lines in terms of the interaction of spin with magnetic moment.

(ii) The yellow line of sodium atom is split into two lines having wavelengths 589.0 nm and 589·6 nm because of the transitions $3P_{\frac{3}{2}} \rightarrow 3S_{\frac{1}{2}}$ and $3P_{\frac{1}{2}} \rightarrow 3S_{\frac{1}{2}}$ respectively. Find the value of the effective magnetic field on the outer electron, which causes this splitting. [Express in terms of Bohr Magneton.] 2+3=5

(d) (i) Write the statistical interpretation of the wave function $\Psi(x, t)$.

(ii) Show with the help of the Schrödinger equation that the space-integrated probability is independent of time. What is the significance of this result so far as the normalization of the wave function is concerned. 1+4=5

3. Answer *any two* of the following questions: 10×2=20

(a) (i) Write down Schrödinger equation for the electron of hydrogen atom assuming the nucleus to be stationary. By separation of the variables, obtain the radial equation.

(ii) The normalized wave function of the ground state of the hydrogen atom is given by

$$\Psi(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-\frac{r}{a_0}}$$

Find the distance from the nucleus at which the electron is most likely to be found.

(iii) Argue from uncertainty principle that only one component of the angular momentum of the electron should be quantized. 5+3+2=10

(b) (i) Write down the Schrödinger equation for a linear harmonic oscillator having mass m, force constant k and frequency ν.

(ii) Obtain the energy eigenvalues and eigenfunctions of the oscillator using the Frobenius method. 1+9=10

(c) (i) Using vector model determine the possible terms corresponding to the principal quantum numbers $n = 3$ and compute the angle between $\vec{l}$ and $\vec{s}$ vectors for the term $2D_{\frac{5}{2}}$.

(ii) Given that the energy of a hydrogen like atom taking into account fine structure corrections is of the form

$$E_{nl} = E_n\left[1 + \frac{Z^2\alpha^2}{n^2}\left(\frac{n}{l+\frac{1}{2}} - \frac{3}{4}\right)\right]$$

where n and l represent the principal and orbital angular momentum quantum number respectively and $\alpha = \frac{1}{137}$. Show that for fixed n the difference between the maximum and minimum energy as l varies from 0 to $n - 1$ is equal to $E_n \cdot \frac{z^2\alpha^2 4(n-1)}{n(2n-1)}$. 6+4=10

(d) (i) What was the purpose of the Stern-Gerlach experiment? Why is a non-uniform magnetic field used in this experiment? In a Stern-Gerlach experiment, always a beam of neutral atoms is used and not ions. — Explain why.

(ii) Elucidate the space quantization of electron spin and describe its demonstration by the Stern-Gerlach experiment. (1+1+1)+(2+5)=10

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